Arnoldi-Lindblad time evolution: Faster-than-the-clock algorithm for the spectrum of (Floquet) open quantum systems

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We introduce a new method to efficiently provide the Liouvillian spectral decomposition. It grants access to the steady state, slow decaying processes and the low-lying spectrum of open quantum systems described by a Lindblad master equation. The method is general and model-independent, applicable to both time-independent systems as well as Floquet systems (i.e. periodically driven). Our method outperforms other diagonalization techniques and retrieves results for systems that would be inaccessible through exact diagonalization.

OPEN SYSTEM TIME EVOLUTION

SYSTEM —coupling— ENVIRONMENT

Lindblad master equation

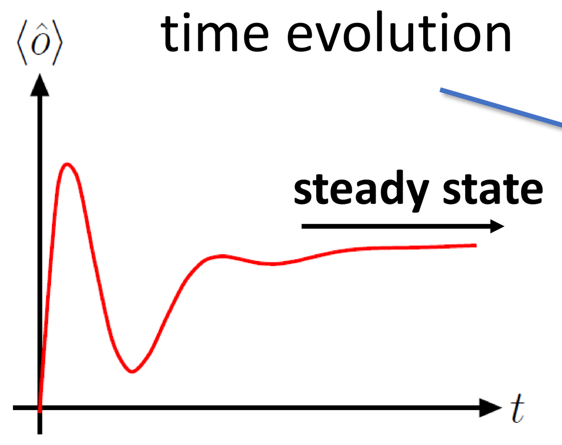
dissipation rate — Lindblad operator "dissipation operator"

$$\partial_t \hat\rho = -i[\hat H, \hat\rho] + \sum_j \frac{\gamma_j}{2}\left(2\hat L_j \hat\rho \hat L_j^\dagger - \{\hat L_j^\dagger \hat L_j, \hat\rho\}\right) = \mathcal{L}[\hat\rho]$$

Liouvillian superoperator

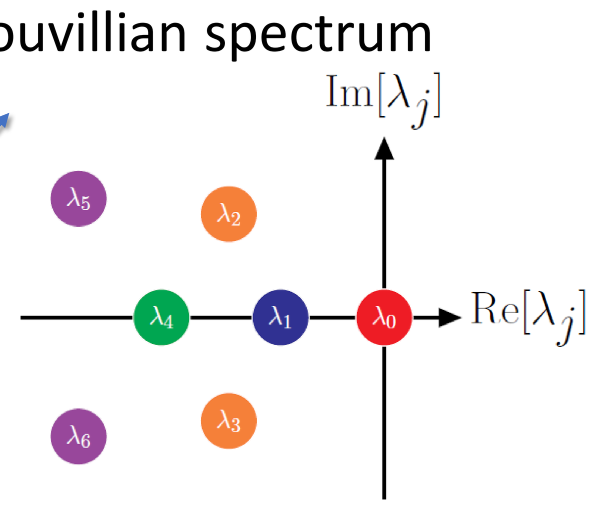
closed system dynamics — coupling with environment (weak and Markovian)

$$\mathcal{L}\hat\rho_j = \lambda_j \hat\rho_j$$



$$\hat\rho(t) = e^{\mathcal{L}t}\hat\rho(0) = \sum_j c_j e^{\lambda_j t}\hat\rho_j$$

Liouvillian spectrum



main interest in "slow" states e.g. the steady state

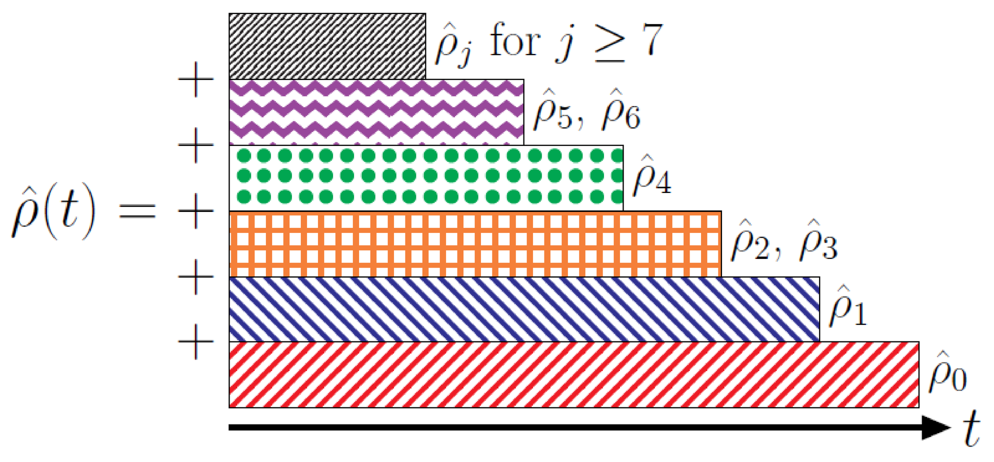
Liouvillian eigendecomposition highly desired

exact diagonalization often infeasible while time evolution is still accessible

THE METHOD

Time evolution

$$\hat\rho(t) = \hat\rho_{SS} + \sum_{j\geq 1} c_j e^{\lambda_j t}\hat\rho_j$$



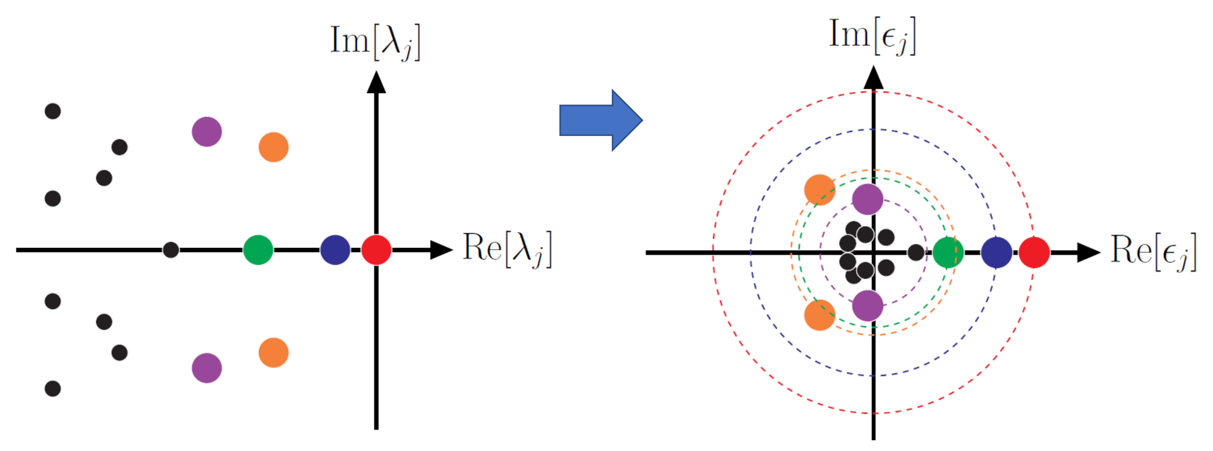
As time passes, the eigenvectors of which the eigenvalues have a large negative real part become negligible in the time evolution.

Spectrum transformation

define a time step T

$$\mathcal{E} = e^{\mathcal{L}T}$$

$$\hat\rho(t) = \left[e^{\mathcal{L}T}\right]^N \hat\rho(0) = \mathcal{E}^N\hat\rho(0)$$



$$\mathcal{E}\hat\rho_j = \epsilon_j\hat\rho_j = e^{\lambda_j T}\hat\rho_j \Leftrightarrow \mathcal{L}\hat\rho_j = \lambda_j\hat\rho_j$$

Krylov and Arnoldi method

$$K_n = \{\hat\rho(0), \hat\rho(T), \hat\rho(2T), \dots \hat\rho(nT)\} = \{\hat\rho(0), \mathcal{E}\hat\rho(0), \mathcal{E}^2\hat\rho(0) \dots, \mathcal{E}^n\hat\rho(0)\}$$

Arnoldi iteration — Gram-Schmidt orthonormalization

$$\mathcal{E}_n^{\text{eff}} = \begin{bmatrix} e_{1,1} & e_{1,2} & e_{1,3} & \cdots & e_{1,n} \\ e_{2,1} & e_{2,2} & e_{2,3} & \cdots & e_{2,n} \\ 0 & e_{3,2} & e_{3,3} & \cdots & e_{3,n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & e_{n,n-1} & e_{n,n} \end{bmatrix}$$

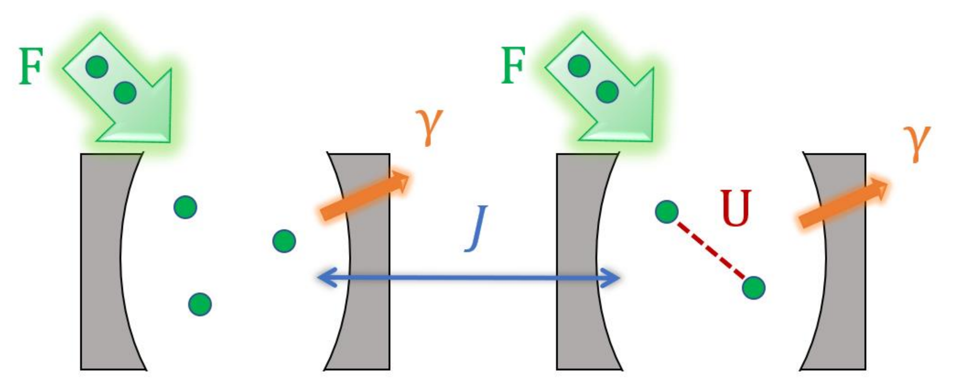
check convergence of eigendecomposition

$$\left\|\mathcal{E}\hat\rho_j^{eff} - \epsilon_j^{eff}\hat\rho_j^{eff}\right\| < \tau$$

$$\mathcal{L}\hat\rho_j^{eff} = \lambda_j^{eff}\hat\rho_j^{eff}$$

THE DRIVEN DISSIPATIVE BOSE HUBBARD DIMER

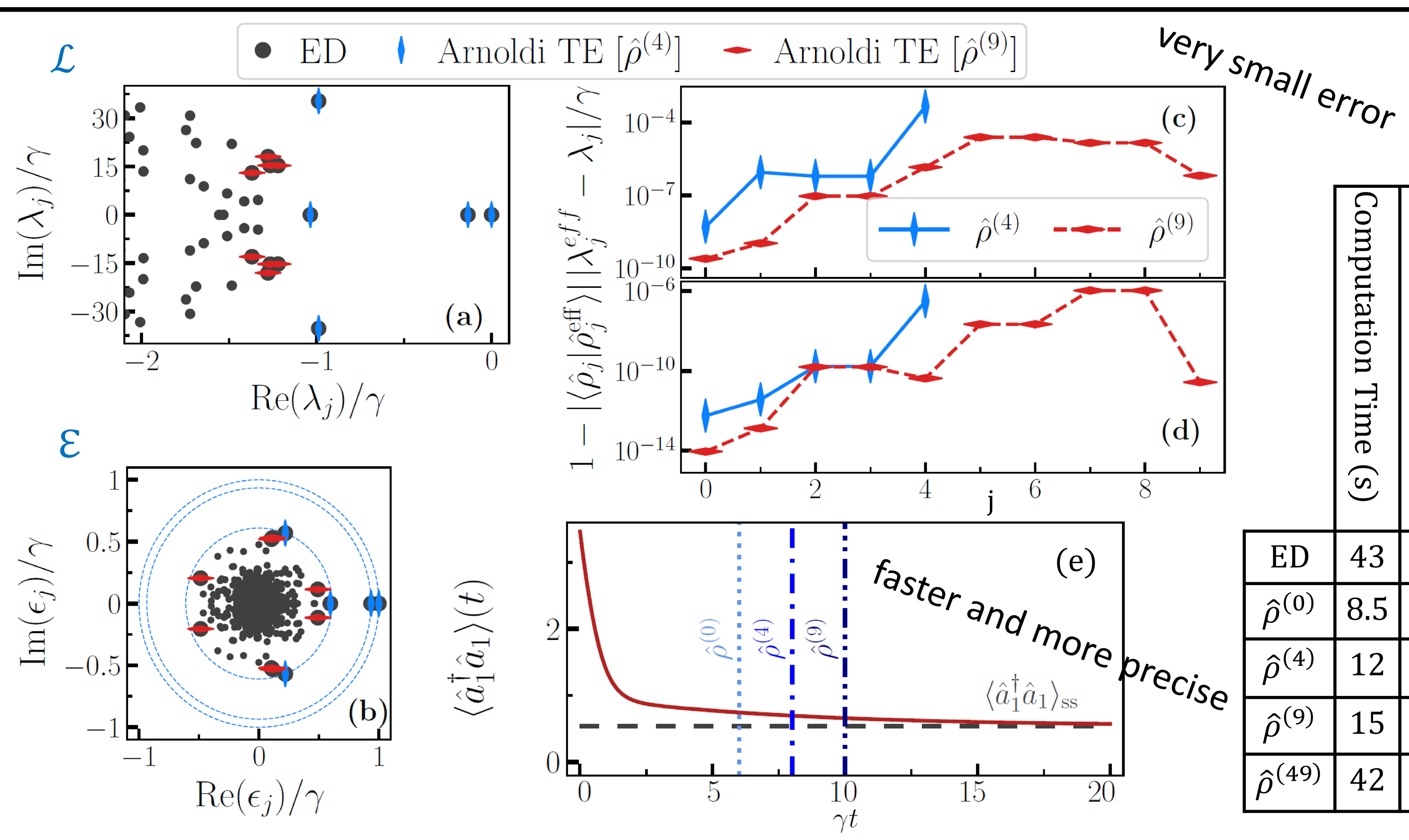
$$\hat H = \sum_{j=1}^{2}\left[-\Delta \hat a_j^\dagger \hat a_j + \frac{U}{2}\hat a_j^\dagger \hat a_j^\dagger \hat a_j \hat a_j + F(\hat a_j^\dagger + \hat a_j)\right] - J(\hat a_1^\dagger \hat a_2 + \hat a_2^\dagger \hat a_1)$$



$$\partial_t\hat\rho = -i[\hat H, \hat\rho] + \frac{\gamma}{2}\sum_{j=1}^{2}\left(2\hat a_j\hat\rho\hat a_j^\dagger - \{\hat a_j^\dagger \hat a_j, \hat\rho\}\right)$$

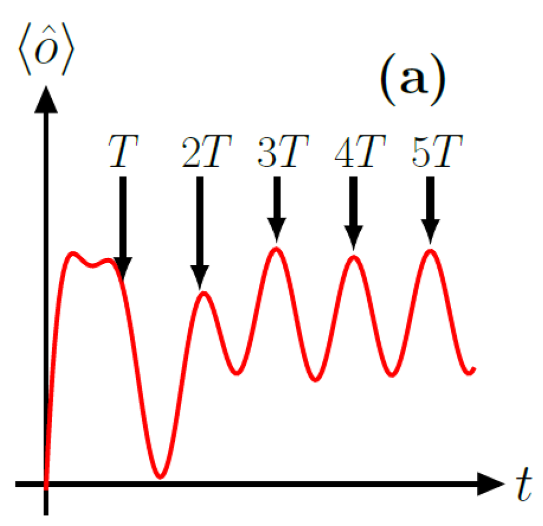
parameters:

$U = 20\gamma$, $F = 4{,}5\gamma$, $n_{max} = 7$, $\Delta = 5\gamma$, $J = 10\gamma$



	Computation Time (s)	Evolution Time (1/γ)	Dim(ℒ^eff)
ED	43		4096
$\hat\rho^{(0)}$	8.5	6	120
$\hat\rho^{(4)}$	12	8	160
$\hat\rho^{(9)}$	15	10	200
$\hat\rho^{(49)}$	42	14	280

FLOQUET SYSTEMS



$$\partial_t\hat\rho(t) = \mathcal{L}(t)\,\hat\rho(t), \qquad \mathcal{L}(t+T) = \mathcal{L}(t)$$

$$\hat\rho(T) = \mathcal{E}\hat\rho(0) \equiv \mathcal{F}\hat\rho(0) = e^{\mathcal{L}^F T}\hat\rho(0)$$

"Floquet map"

stroboscopic eigenspectrum

$$\mathcal{F}\hat\rho_j^F = \varphi_j^F\hat\rho_j^F \Updownarrow \mathcal{L}^F\hat\rho_j^F = \lambda_j^F\hat\rho_j^F$$

CONCLUSIONS

- Method to efficiently determine the low-lying eigenvalues and eigenmatrices of the evolution operator/ Liouvillian;
- Retrieves these features through a (shorter) time evolution of the open system;
- Grants access to eigendecomposition of bigger system sizes;
- Outlook: extension to approximated time evolutions of open quantum systems as well as the quantum trajectory formalism.

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