Kasteleyn transition in a Coulomb phase

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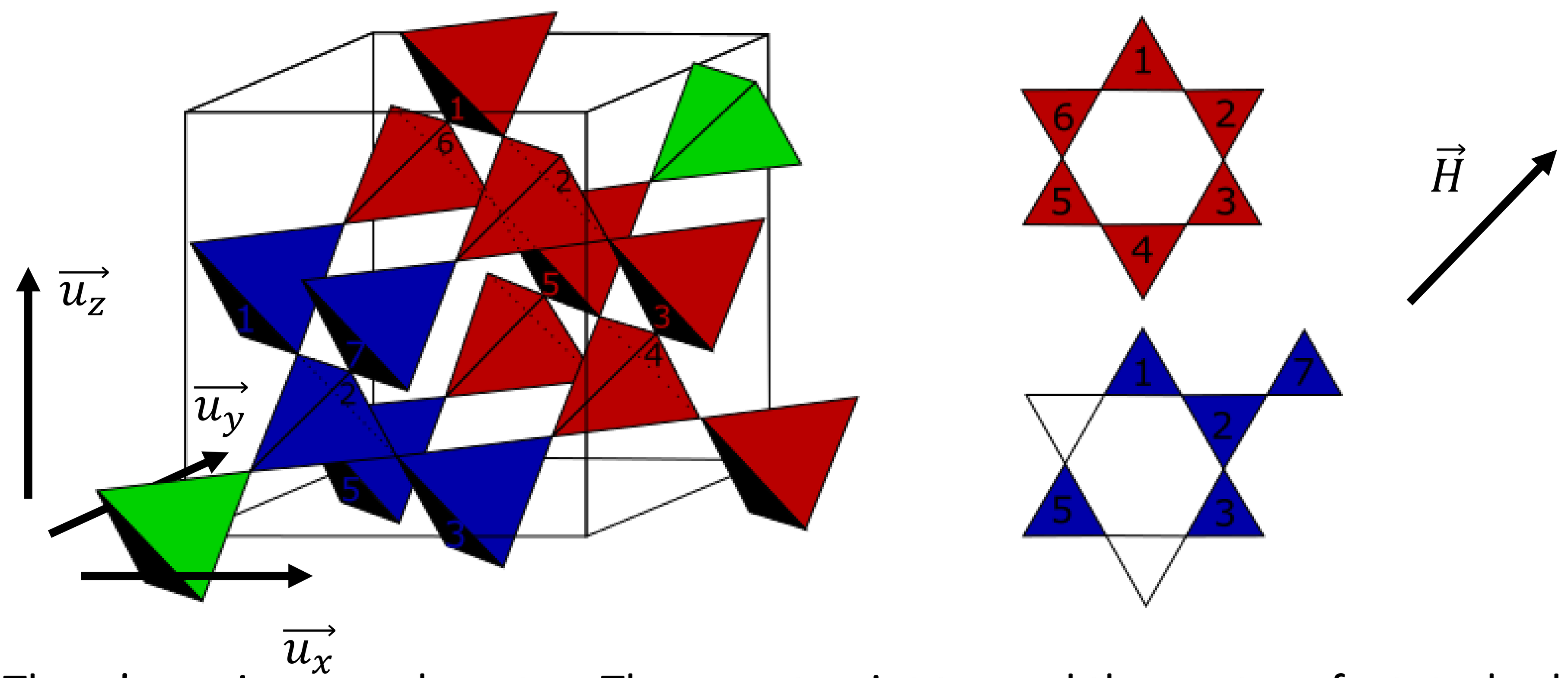
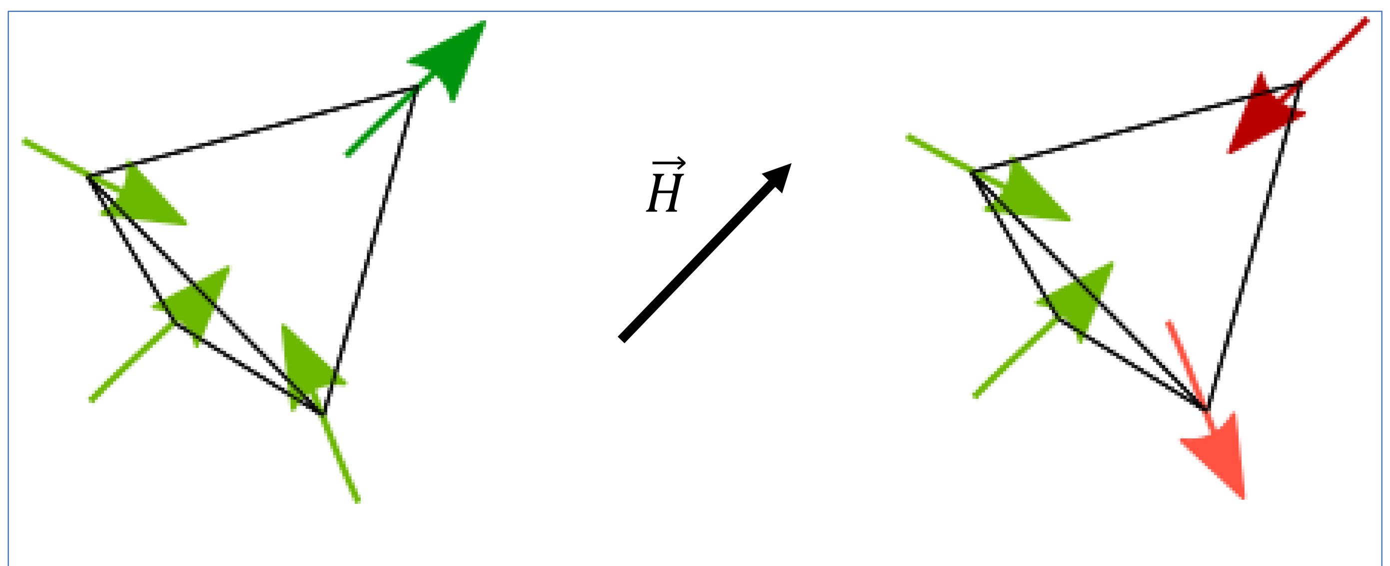
We apply a magnetic field along the [111] direction for spin ice in the monopole crystal phase - a strongly fragmented phase in which a longitudinal fragment of the magnetic moments show antiferromagnetic long range order characteristic of a monopole crystal while the remainder, the transverse part forms a Coulomb phase with dipolar correlations [1,2,3]. The magnetic field couples to the transverse component, driving it towards saturated ferromagnetic state via a Kasteleyn transition [4].
We present numerical results generated from a worm Monte Carlo algorithm confirming the Kasteleyn transition. Tilting the field away from the [111] direction, the system orders in one of four topological sectors over a sphere of solid angle 4pi.
We show that the specific heat and susceptibility diverge logarithmically at the transition, consistently with the system being at the lower critical dimension. The reduced symmetry for the Coulomb phase correlations due to the applied field shows up in the evolution of simulated neutron scattering plots as the transition is approached. These show pinch point patterns that are continuously deformed as the field strength grows.



- There's a spin on each vertex. They must point toward the centre of a tetrahedron.
- The odd tetrahedrons must **always** have three spins in, and even ones three spins out.



- T_c verifies $0 = \Delta F = \Delta E - T_c \Delta S$

where $E = -\vec{S}.\vec{H}$ and $S = -\sum p \ln p$

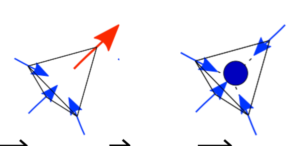
- We obtain an equation for T_c: $e^{\mu_0} = e^{\mu_1} + e^{\mu_2} + e^{\mu_3}$

where μ_i are the chemical affinities $\mu_i = e^{-2\vec{S_i}.\vec{H} / k_B T_c}$

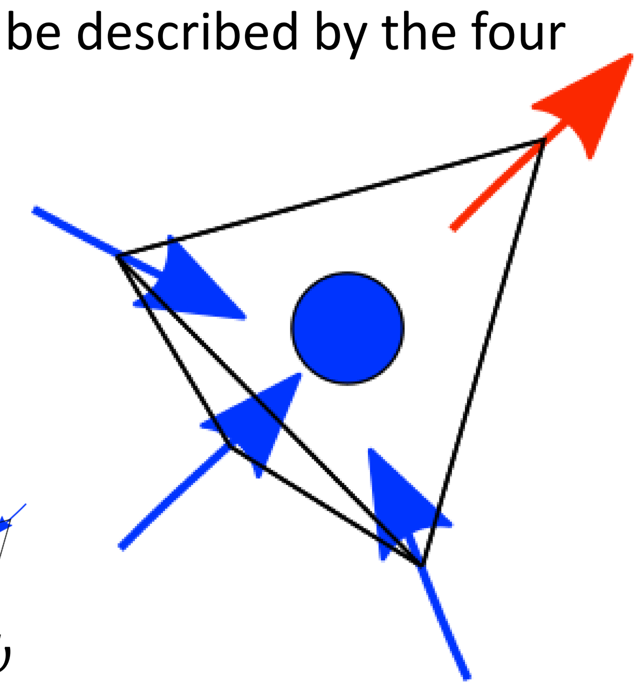
- The magnetisation field, $\vec{M}$ of a given tetrahedron can be described by the four spins around.
- For instance, here : $\vec{M} = [+1, -1, -1, -1]$
- The field can then be decomposed in two terms:

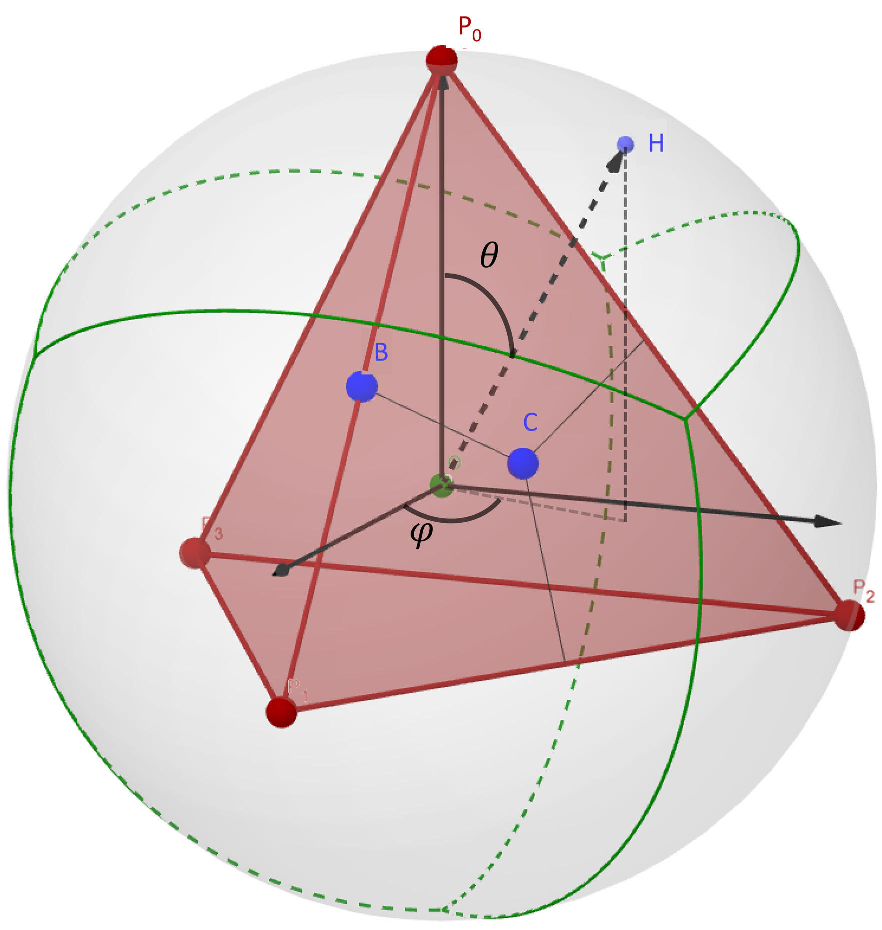
$$\vec{M} = \left[\frac{3}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}\right] + \left[\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}\right]$$

- Following the Helmholtz Hodge theorem we have:
  - A divergence less term (crystal phase)
  - A rotational less term (Coulomb phase)

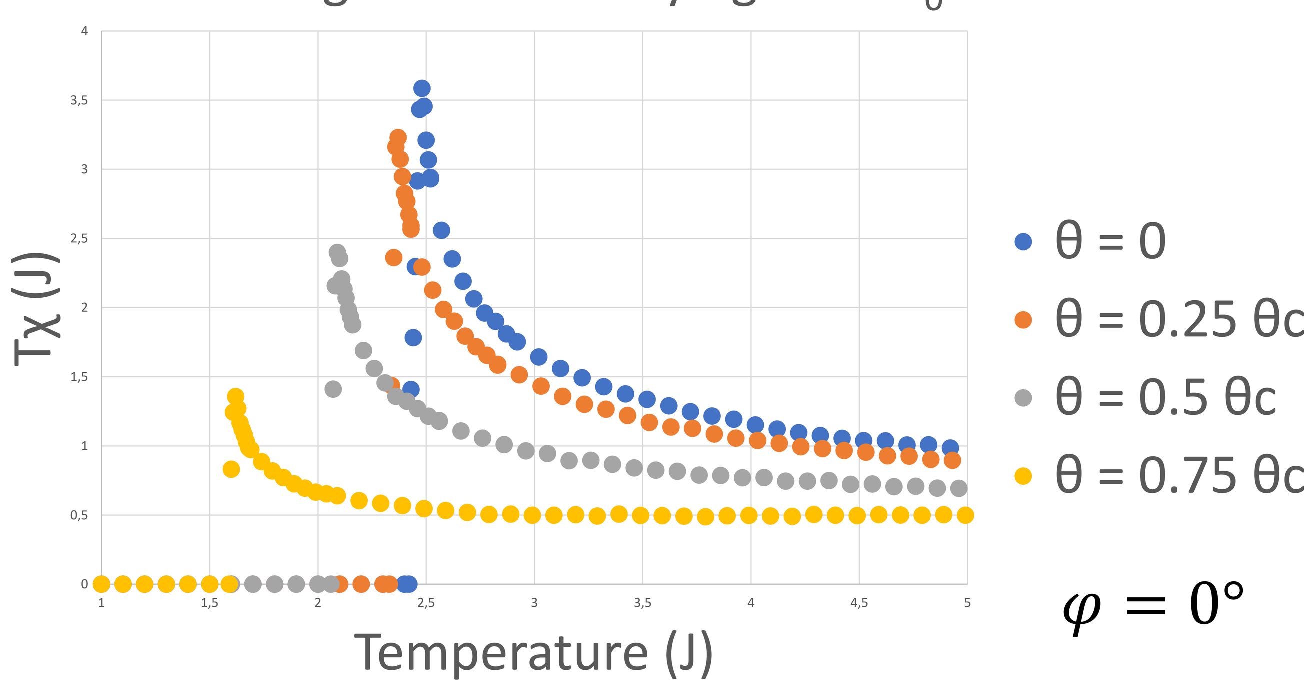


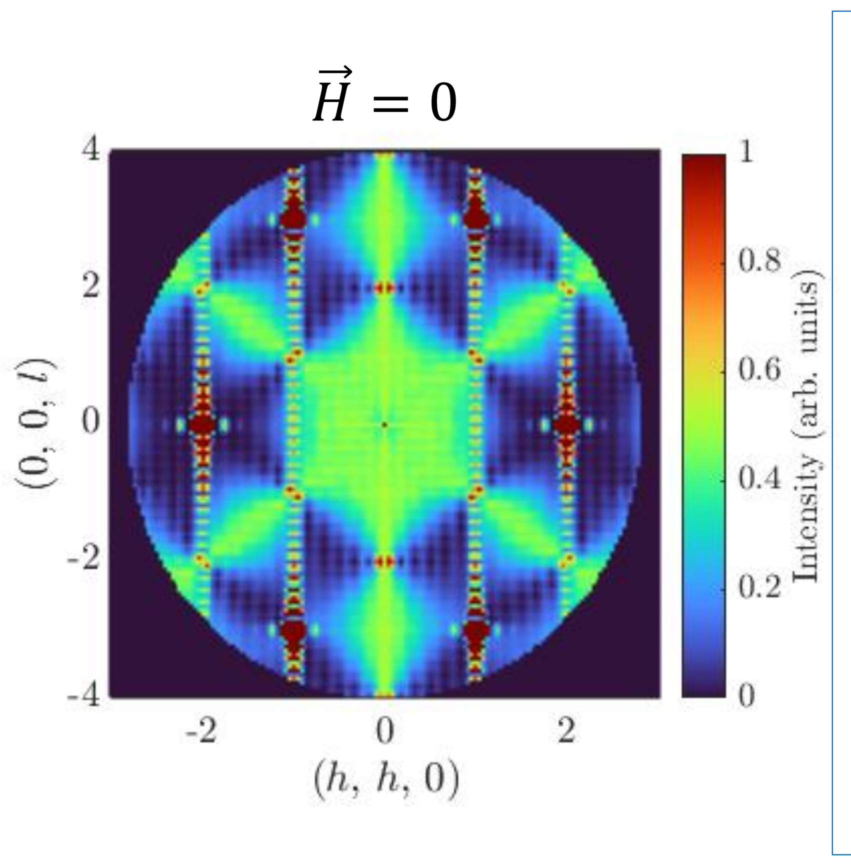
- We can then rewrite magnetisation: $\vec{M} = -\vec{\nabla} \times \vec{A} + \vec{\nabla}\psi$



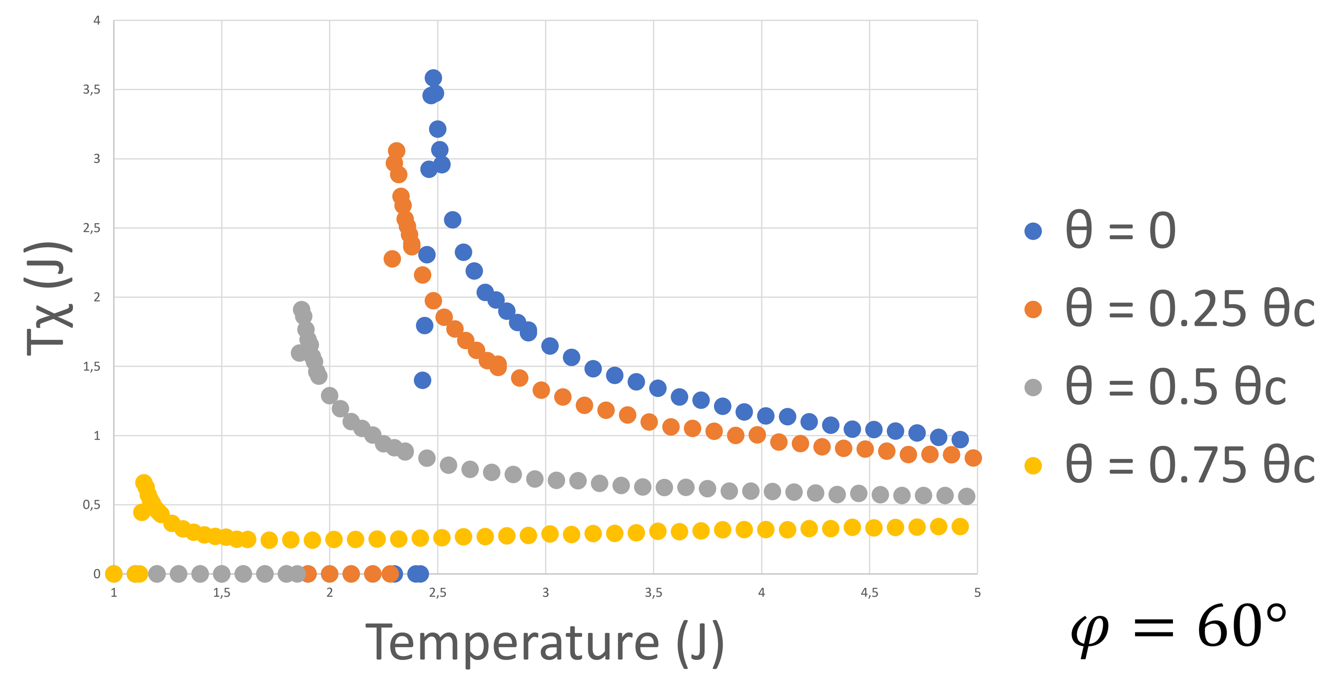


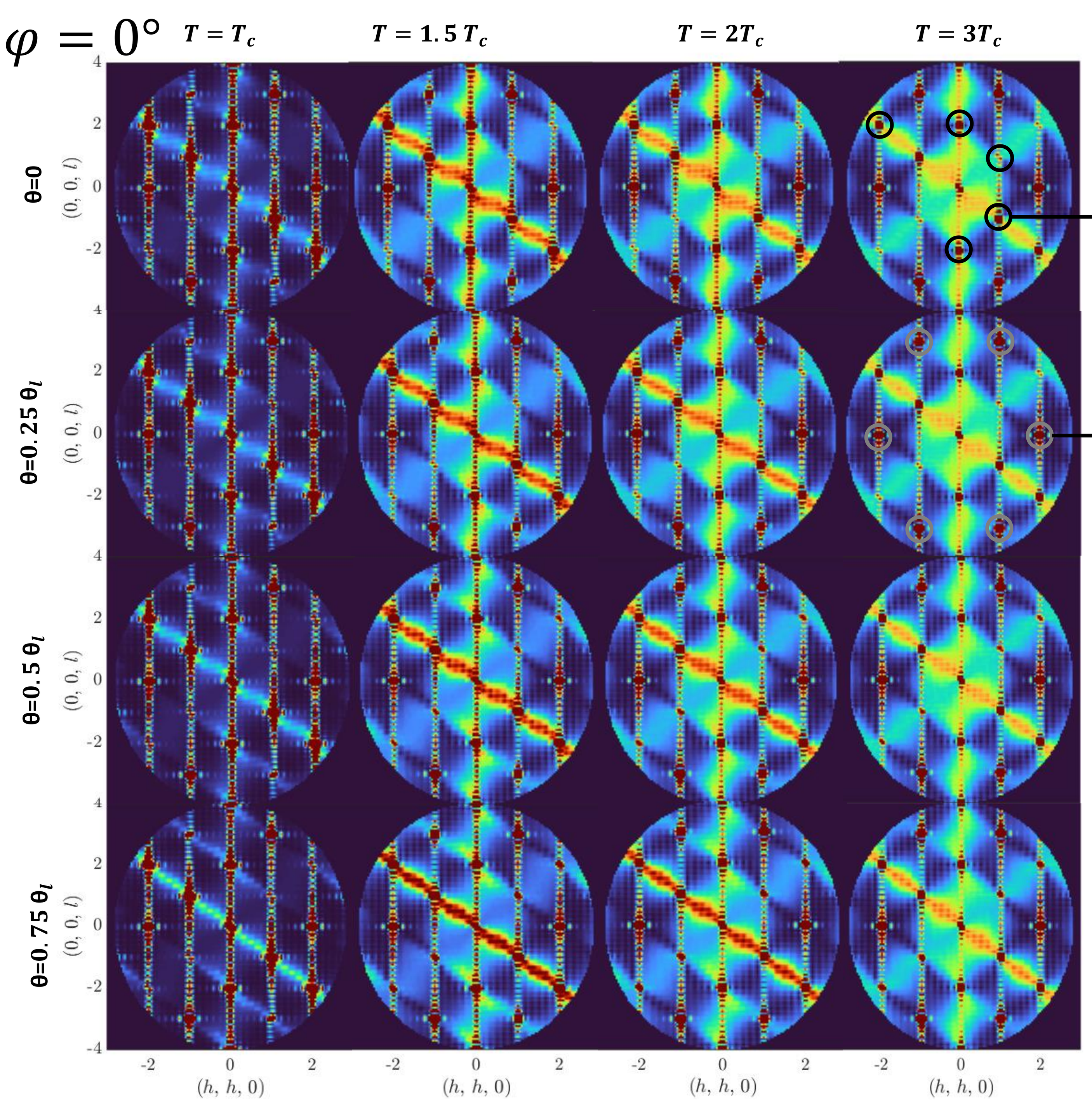
Magnetic susceptibility evolution for a magnetic field varying from P_0 to B

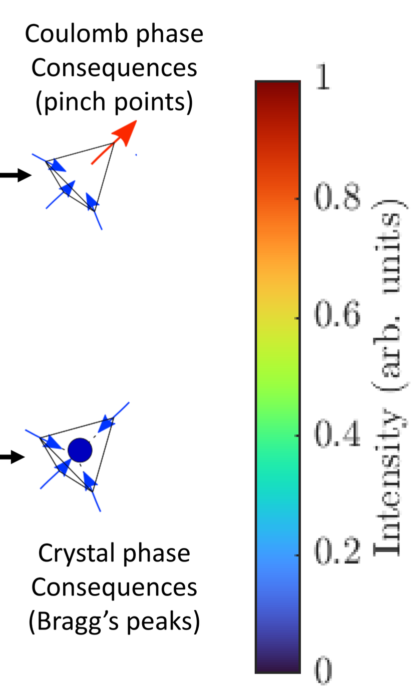


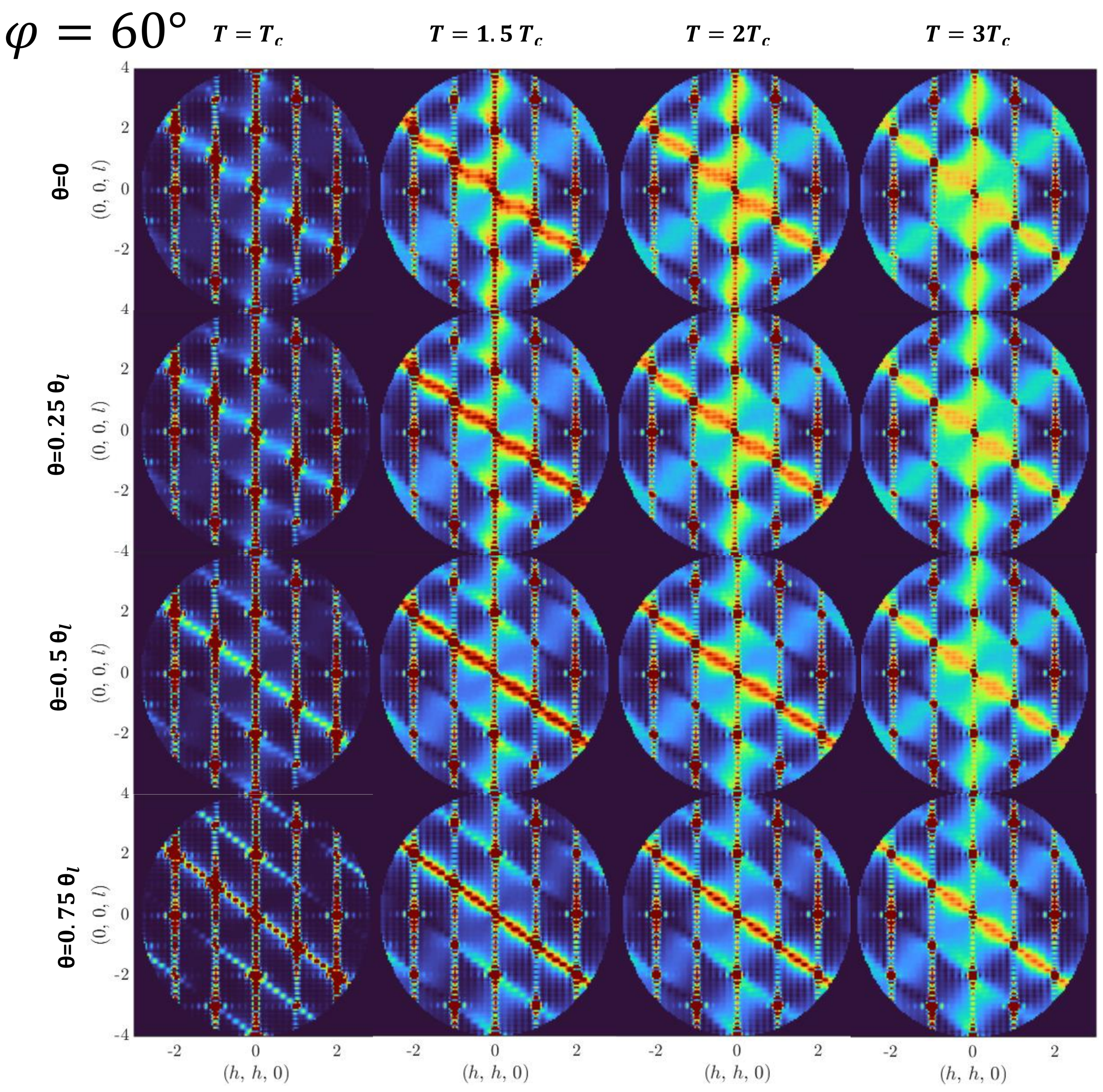


Magnetic susceptibility evolution for a magnetic field varying from P_0 to C









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